Answer Sheet to the Written Exam
Corporate Finance and Incentives

February 2019

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "After completing the course, the student should be able to:

Knowledge:

1. Understand, account for, define and identify the main methodologies, concepts and topics in Finance
2. Solve standard problems in Finance, partly using Excel
3. Criticize and discuss the main models in Finance, relating them to current issues in financial markets and corporate finance

Skills:

1. Manage the main topics and models in Finance
2. Organize material and analyze given problems, assessing standard models and results
3. Argue about financial topics, putting results into perspective, drawing on the relevant knowledge of the field

Competencies:

1. Bring into play the achieved knowledge and skills on new formal problems, and on given descriptions of situations in financial markets or corporations
2. Be prepared for more advanced models and topics in Finance."

Problems 1–3 are particularly focused on knowledge points 1 and 2, skills of type 1 and 2, competencies 1 and 2. Problem 4 emphasizes knowledge points 1 and 3, skills 1 and 3, and competency 1.

Some numerical calculations may differ slightly depending on the commands chosen for computation, so a little slack is allowed when grading the answers.

Problem 1 (APT 25%)

1) Define

$$\alpha = \begin{bmatrix} 0.165 \\ 0.049 \\ 0.125 \\ 0.052 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.7 & 0.5 & 1.2 & 1 \\ -0.3 & 0.9 & 0.4 & 1 \\ 0.8 & -0.2 & 1.1 & 1 \\ 0.1 & 0.6 & -0.4 & 1 \end{bmatrix}.$$

Portfolio x should solve $x^T B = (0,0,0,1)$, so $x^T = (0,0,0,1)\,B^{-1}$. Use matrix inversion in Excel to find $x^T = (-1.07, 0.57, 1.10, 0.39)$. The risk-free rate is $x^T \alpha = 0.01$, i.e., $r_f = 1\%$.

2) The first pure factor portfolio is, transposed, $(1,0,0,1)\,B^{-1} = (0.46, -1.01, 0.31, 1.24)$. The second pure factor portfolio is, transposed, $(0,1,0,1)\,B^{-1} = (0.56, 0.26, -0.47, 0.65)$. The third pure factor portfolio is, transposed, $(0,0,1,1)\,B^{-1} = (-0.91, 1.15, 1.29, -0.53)$.

3) The first pure factor portfolio's expected return is $(0.46, -1.01, 0.31, 1.24)\,\alpha = 13\%$. Its risk premium is $\lambda_1 = 13\% - 1\% = 12\%$. The second pure factor portfolio has expected return 8%; risk premium $\lambda_2 = 7\%$. The third one has expected return 4%; risk premium $\lambda_3 = 3\%$.

4) In the absence of arbitrage, the expected return on this asset should be $r_f + (1,1,1)\,\lambda = 23\%$, but it is actually 21%. So, arbitrage is possible. It can, for instance, be exploited by purchasing one of each of the three pure factor portfolios (corresponding to the factor weights $(1,1,1)$), while selling the new, fifth asset.

Problem 2 (Corporate Governance 25%)

1) The expected present value of project 1 is $.4\,(100) - .6\,(40) = 16$, and here the Founder gets private benefit 27. For project 2 this expected present value is $.7\,(100) - .3\,(40) = 58$, and there is no private benefit. The two claims are then correct, as $58 > 43 = 16 + 27$ and $.6\,(58) = 34.8 < 36.6 = .6\,(16) + 27$.

2) Given that the Founder chooses project 1, the outside investors will get $.4\,(16) = 6.4$.

3) Since $.7\,(58) = 40.6 > 38.2 = .7\,(16) + 27$, the Founder now prefers project 2.

4) The Founder is willing to pay the payoff difference $40.6 - 36.6 = 4$. This 10% stake was worth $.1\,(16) = 1.6$ to outside investors, which is less than 4.

5) With the new project choice, a 10% stake is worth $.1\,(58) = 5.8$, exceeding 4.

Problem 3 (Options 25%)

1) This is explained on page 775 in Berk and DeMarzo.

2) This is also explained on page 775 in Berk and DeMarzo.

3) In the five states, starting from Very Bad, the call option will be worth: $0, 0, 0, 18, 62$. For example, consider the Very Good state. From 1), the option is best exercised before the dividend is paid. At that stage, the stock's value is $120 + 12 = 132$. The option allows to obtain this value by paying 70, so the gain is $132 - 70 = 62$. Using the risk-neutral probabilities and the risk-free rate, in today's market the call option is worth

$$\frac{.15\,(0) + .25\,(0) + .35\,(0) + .2\,(18) + .05\,(62)}{1.03} = 6.50.$$

4) In the five states, starting from Very Bad, the put option will be worth: $60, 40, 20, 0, 0$. For example, consider the Very Bad state. From 2), this option is best exercised after the dividend is paid. At that stage, the stock is worth 10 in the market. The option allows to get paid 70 for this, so the gain is $70 - 10 = 60$. Here, the premium is 25.24.

5) In general, the Put-Call parity can be expected to fail when American options are exercised at different points in time. On page 770, Berk and DeMarzo present a put-call parity for European options on a dividend-paying stock, $C = P + S - PV\,(Div) - PV\,(K)$ in their notation. In the problem, $PV\,(Div) = 3.83$ and $PV\,(K) = 67.96$. The future payoffs to the stock will be $10, 30, 55, 88, 132$, worth $S = 50.92$. With $C = 6.50$ and $P = 25.24$ from 3) and 4), this parity fails for these American options.

Problem 4 (Various Themes 25%)

1) From page 531 in Berk and DeMarzo, in their notation, the proposition predicts $r_E = r_U + (D/E)\,(r_U - r_D)$. This implies the desired equation as explained on pages 534–535.

2) The first part of the text suggests that the risk-premium paid on risky assets has risen. In the CAPM and APT models, this could simply be the market risk premium that has risen. The key question is then why this happens in a situation where the FED views the economy as doing well. One suggestion could be that investors are panicking, and not really using rational expectations. Berk and DeMarzo discuss the implications of rational expectations on page 481, and the remainder of their chapter 13 connects this to efficiency of the market portfolio and the performance of the CAPM and APT models. It is also possible that the FED is mistaken about the outlook, and that markets know better. The last part of the text suggests that either the FED or the market might soon have reason to change views.

3) This option should satisfy the put-call parity $C = P + S - PV\,(K)$, where $P \geq 0$ denotes the corresponding put premium. With positive risk-free interest, $PV\,(K) < K$. Using these two inequalities, the parity delivers $C = P + S - PV\,(K) > S - K$.